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PE and PEG both matter: a simple formula for unifying them

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Abstract

In business valuation reports, multiples computed by dividing the price of a comparable company's stock by some relevant accounting numbers (a measure of income for example) are frequently used. Two ratios are especially very popular, the PE and PEG ratios. We show through a limited analytical development that generally no one ratio (PE or PEG) captures all the information provided by the accounting data (forecasted earning and expected growth) and it might be more accurate to combine both of them into a single approach. The proposed model is part of the literature of current valuation models based on accounting drivers, such as RIM or AEG models. Its goal is to take into account the effects of short-term growth, as generally forecast by financial analysts. It provides a simple extension to the so-called traditional model DDM and can be used to infer an estimate of the implied cost of capital.

Introduction

In business valuation reports, multiples computed by dividing the price of a comparable company's stock by some relevant economic variables (Liu et al., 2002, Pratt et al., 2000 for example) are frequently used. The most popular value drivers are accounting measures (most often forecast incomes). The link between the accounting numbers and the firm's value is also frequently found in theoretical developments. Both accounting and finance literature propose several firm valuation models based on accounting items. The oldest, best-known and most frequently cited is without doubt the Gordon-Shapiro model. During the nineties, Ohlson (1995), Feltham and Ohlson (1996) developed new models highlighting in the valuation formulas the importance of the book value and the income measure. These models are known as the Residual Income Models (RIM). Later, Ohlson and Juettner-Nauroth (2004) put forward the Abnormal Earning Growth model (AEG), using data from the expected growth of income. The use of these two types of models does not seem to have gone beyond the confines of empirical studies conducted in academic circles. They are generally absent from reports by practitioners. It is true that each appears to generate specific and very pronounced biases: the first systematically undervalues companies and the second overestimates them. In this short paper, we show through a limited analytical development that generally no one ratio (PE or PEG) captures all the information provided by the accounting data and it may be more reasonable in an empirical context to combine both of them into a single approach.

Hypotheses

The two basic points behind this proposed model are:

- drawing on Ohlson, we keep the idea that the ability of a company to create value from in-house activities is not endless but tends to reduce, due to competition and innovation
- we introduce the supplementary idea that, when growing, the company seeks to generate new value-creating opportunities. Retained earnings finance new positive NPV projects.

From these two assumptions, we show that it is possible to derive a model where the expected income grows at a variable speed: the growth rate is of a mean reverting type, with a long-term target. To facilitate the search for an analytical solution, the model is developed in continuous time.

As is often the case in this kind of model, the shareholder value of the company is defined as the present value of free cash flows for equities:

Hypothesis 1:

$$V_0 = \int_0^{+\infty} F(t) \times e^{-k \cdot t} dt \qquad (1)$$

k	Cost of equities
p	Abnormal profitability derived from the use of retained earnings
ω	Decreasing rate of the abnormal profitability of the in-house assets
g	Long-term growth rate for the expected incomes
r	Retention rate (1 – pay-out ratio)
B(t)	Book value of equities
F(t)	Free cash-flow for equities at time t
X(t)	Net income at t

Table n°1: notations

Conceptual framework:

As is usual in practice, the model is anchored to a measure of accounting income. From the point of view of the shareholders, it is the expected net profit. Our forecast is mainly based on three parameters: the current profitability of the assets in place, the company's ability to maintain this level of profitability, and finally the ability of the company to generate profitable growth through new opportunities. Formally, Hypothesis 2 follows the lines of the conceptual framework of RIM models with the additional introduction of growth opportunities. Here lies its originality.

Hypothesis 2:

- Following Ohlson and others, the abnormal return on current assets is assumed to decrease at a constant negative rate ω: competition and innovation erode the "rents" in place.
- Retained earnings finance value-creating opportunities with an abnormal profitability p. The firm retains an ability to innovate.

$$X'(t) = \omega \times [X(t) - k \times B(t)] + B'(t) \times [k + p] \qquad (2)$$

The forecast profitability of the company is driven by two forces: one pushing down (ω <0), the other upwards. The second factor is not, as in Ohlson, a positive exogenous short-term shock but is linked to the company's ability to generate value-creating opportunities.
To complete the analysis, we have to express how capital will accumulate within the company. In reality, the phenomenon is complex because it depends as much on the environment, the ability of the company to generate new opportunities and its organizational constraints. To maintain the viability of the model, we choose to make a simplifying assumption. Growth in this hypothesis is limited by the company's ability to raise capital. The only source of new capital is in the retention of a constant share of the incomes. This third hypothesis has the advantage of taking into account a constraint sustainable over the long term, but which is too simplistic in the short term.

<u>Hypothesis 3</u>:
We assume that the company maintains a constant ratio of retention, sufficient for financing long-term sustainable growth:

$$1-\frac{F(t)}{X(t)}=r \qquad (3)$$

As usual in this kind of literature, the accounting framework is assumed to satisfy the following hypothesis.

<u>Hypothesis 4</u>:

$$B'(t)=X(t)-F(t) \qquad (4)$$

This last assumption is the so-called "clean surplus" condition.

The chosen dynamics for the income variable:

Because the forecast income is the crucial variable in this model, it is important to understand the impact of these assumptions on the dynamics envisaged. Differentiating (2) and introducing (3) and (4), we obtain:

$$X''(t)=X'(t)\times[\omega+r\times(k+p)]-X(t)\times\omega\times r\times k \qquad (5)$$

Following the assumptions, the growth of net income of the company appears as mean-reverting. Recall that the differential equation of a mean-reverting process representing a flow X (t) is of the form:

$$X''(t)=-\delta.[X'(t)-\gamma_1.X(t)] \qquad (5‘)$$

with $\gamma_1=g.\left(1+\frac{g}{\delta}\right)$ where g a long-term growth rate and δ the adjustment speed of the growth of the output to its target rate.
The selected framework distinguishes a growth rate in the short term and one also for the long term, while keeping a steady convergence towards one another. Note that from the comparison of (5) and (5'), we can derive the following equations:

$$\delta=-[\omega+r\times(k+p)] \qquad (6\text{-}1)$$

There are two ways of modelling this: either r is set exogenously and g is determined by the previous equality, or g is an exogenous parameter and a sustainable ratio of retention r is derived from the following equation:

$$r=g\times\frac{g-\omega}{g\times(k+p)-\omega\times k} \qquad (6\text{-}2)$$

In (6-2) and in the further development, we choose the second path, following the standards of the literature as with the Gordon-Shapiro model.
Note that (6-2) can also be rewritten as:

$$r=\frac{g}{k}\times\frac{1}{1+\frac{g}{k}\times\frac{p}{g-\omega}}$$

The retention ratio is the product of a sustainable long-run ratio for a company creating no value: $\frac{g}{k}$ and a correction factor taking into account its ability to generate value: $\frac{1}{1+\frac{g}{k}\times\frac{p}{g-\omega}}$

Introducing (6-2) in (6-1), we obtain:

$$\delta=\frac{-g^2\times(k+p)-\omega^2\times k}{g\times(k+p)-\omega\times k} \qquad (6\text{-}3)$$

For the coefficient of the speed of adjustment to be positive, the following condition has to be satisfied:

$$p<k\times\left[\frac{\omega^2}{g^2}-1\right]$$

Given the constraint that $p>0$, it is necessary that -ω> g. This requirement seems reasonable: the erosion factor of the rent is greater than the long-term growth rate.

At this stage, the differential equation (5) has to be solved with the following initial conditions:

$$X(0)=X_0 \qquad (7\text{-}1)$$
$$X'(0)=\gamma_0\times X_0 \qquad (7\text{-}2)$$

The solution (see appendix A) is:

$$X(t)=X_0\times\left[\frac{\frac{g+\gamma_0}{2}+\frac{\delta}{2}}{g+\frac{\delta}{2}}\times e^{g.t}+\frac{\frac{g-\gamma_0}{2}}{g+\frac{\delta}{2}}\times e^{-(g+\delta).t}\right] \qquad (8)$$

It appears that following our assumptions, the future value of the accounting income of the firm is only a function of four parameters:

- X_0 , the instantaneous present flow
- $\gamma_0\times X_0$, its instantaneous present growth
- g , the asymptotic growth rate in the long term
- δ , the adjustment factor for the growth equal to $\frac{-g^2\times(k+p)-\omega^2\times k}{g\times(k+p)-\omega\times k}$

Valuation of the company for its shareholders:

As standard, the shareholder value of the firm is equal to the sum of the expected discounted free cash flows for equities and introducing (3) in (1), we get:

$$P_0 = \int_0^{+\infty} X(t) \times (1-r) \times e^{-k \cdot t} dt \qquad (9)$$

Introducing (8) in (9), we get:

$$P_0 = \int_0^{+\infty} X_0 \times \left[\frac{\frac{g+\gamma_0}{2} + \frac{\delta}{2}}{g + \frac{\delta}{2}} \times e^{g.t} + \frac{\frac{g-\gamma_0}{2}}{g+\frac{\delta}{2}} \times e^{-(g+\delta).t} \right] \times (1-r) \times e^{-k \cdot t} dt \qquad (10)$$

Or:

$$P_0 = X_0 \times (1-r) \times \left[\frac{\frac{g+\gamma_0}{2} + \frac{\delta}{2}}{g + \frac{\delta}{2}} \times \frac{1}{k-g} + \frac{\frac{g-\gamma_0}{2}}{g+\frac{\delta}{2}} \times \frac{1}{k+g+\delta} \right]$$

After simplification, we can write (11):

$$P_0 = \frac{X_0 \times (1-r)}{k-g} \times \left[1 + \frac{\gamma_0 - g}{k+\delta+g} \right]$$

with

$$r = \frac{g}{k} \times \frac{1}{1 + \frac{g}{k} \times \frac{p}{g-\omega}}$$

$$\delta = \frac{-g^2 \times (k+p) - \omega^2 \times k}{g \times (k+p) - \omega \times k}$$

This model appears as an extension of the traditional DDM. It has the advantage of incorporating not only the expected income but also its short-term growth rate (γ_0) and its ability to create value (p). The key point is the identification of a corrective factor: $\left[1 + \frac{\gamma_0 - g}{k+\delta+g} \right]$.

Note that the model degenerates into $P_0 = \frac{X_0}{k} \times \left[1 + \frac{\gamma_0 - g}{k-\omega} \right]$ if $p=0$ and $P_0 = \frac{X_0}{k}$ if $\gamma_0 = p = 0$. Finally, it appears that this model contains more information than those which put emphasis only on the present profitability (X_0).

Continuous time modelling has obvious advantages in terms of derivation but is unsuited to the practical framework where forecasts are generally produced on a discrete and annual basis. We can transcribe the model into discrete time.

For simplicity, we will refer to a measure of the annual net income, NI_t . Note first that the expected net income for the year t can be written as:

$$\mathrm{NI}_t=\int_{t-1}^{t} X_0\times\left[\frac{\frac{g+\gamma_0}{2}+\frac{\delta}{2}}{g+\frac{\delta}{2}}\times e^{g.t}+\frac{\frac{g-\gamma_0}{2}}{g+\frac{\delta}{2}}\times e^{-(g+\delta).t}\right]dt$$

This expression is based on the two variables X_0 and $\gamma_0\times X_0$. It is easy to express these last two quantities in terms of two measures of the expected annual income. For example, taking into account the most readily available forecasts (one year and two), we can write (see Appendix B):

$$X_0=m_{11}(g,\delta)\times \mathrm{NI}_1+m_{12}(g,\delta)\times\Delta\,\mathrm{NI}_2$$
$$X_0\times\gamma_0=m_{21}(g,\delta)\times \mathrm{NI}_1+m_{22}(g,\delta)\times\Delta\,\mathrm{NI}_2$$

The model (11) allows us to highlight the specific factor related to the short-term growth:

$$\left[\frac{\gamma_0-g}{k+\delta+g}\right]=\frac{\left(\frac{m_{11}(g,\delta)\times \mathrm{NI}_1+m_{12}(g,\delta)\times\Delta\,\mathrm{NI}_2}{m_{21}(g,\delta)\times \mathrm{NI}_1+m_{22}(g,\delta)\times\Delta\,\mathrm{NI}_2}-g\right)}{(k+\delta+g)}$$

	Basis	**Decrease**	**Growth**	**Strong growth**	**Very strong growth**
Short term variation in the income	2%	-10%	7%	12%	32%
ω =-5%					
p=5%	0%	-79%	36%	73%	245%
p=2%	0%	-77%	35%	71%	238%
ω =-10%					
p=5%	0%	-55%	25%	51%	168%
p=2%	0%	-54%	24%	49%	163%
ω =-30%					
p=5%	0%	-19%	8%	17%	54%
p=2%	0%	-18%	8%	17%	53%

Table 2: Importance of current growth

Table 2 highlights the importance of the short-term growth factor in the valuation of a firm. In this simulation, the values of parameters k and g are respectively 10% and 2%. The results provide evidence of the weight of the factor for different values of both parameters p and ω . When the short-term growth is equal to the long term, the factor is zero. When the short-term growth is strong, for example 12% (or 10% above the long-term growth rate), if the persistence is strong (eg ω equal to -10%), the growth factor is around 50%. The information provided by the expected change in the income ($\Delta\,\mathrm{NI}_2$) is important in the valuation process, together with that contained in the next forecast income (NI_1), (the latter is also affected by the short-term growth: it is the difference between NI_1 and X_0) . Symmetrically, a short-term decline in results has a significant effect on the value, even with a low persistence coefficient (here of about 19% for ω equal to 30%). Finally, the parameter p does not appear to have a significant effect on this factor.

Conclusion:

This model is part of the literature of current valuation models based on accounting drivers, such as RIM or AEG models. Its goal is to take into account the effects of short-term growth, as generally forecast by financial analysts which, as it is not confined to the calculation of a single PEG ratio, is very often unsatisfactory in practice. It provides a relatively simple extension to the so-called traditional model DDM. It aims to avoid systematic effects of undervaluation of RIM models and overvaluation of AEG models. The result is obtained at the cost of introducing two additional parameters p and ω, the second playing a very important role.
As this model allows an assessment based on forecasts of net income and short-term growth, setting the values for the parameters k , p ,ω (specific to the company) and g (specific to the economy), it can be used to look backwards from observed stock values so as to infer an estimate of the implied cost of capital k (see Gebhardt et al., 2001 and Gode and Mohanram, 2003, for example). It can enrich this method of estimating the cost of capital, a useful alternative to the models with one or multiple factors.

Appendix A: Solving the differential equation characterizing the dynamics of the income

The Laplace transform of the function $X(t)$ is $x = L(X(t))$

Noting:

$$L(X'(t)) = s \times x - X_0$$
$$L(X''(t)) = s^2 \times x - s \times X_0 - \gamma_0 \times X_0$$

Introducing the Laplace transforms in the differential equation (5’) :

$$s^2 \times x - s \times X_0 - \gamma_0 \times X_0 + \delta \times s \times x - \delta \times X_0 - \gamma_1 \times \delta \times x = 0$$

We get :

$$x = X_0 \times \frac{s + \gamma_0 + \delta}{s^2 + \delta \times s - \delta \times \gamma_1}$$

or

$$x = X_0 \times \frac{\left(s + \frac{\delta}{2}\right)}{\left(s + \frac{\delta}{2}\right)^2 - \left(\delta \times \gamma_1 + \frac{\delta^2}{4}\right)} + X_0 \times \left(\gamma_0 + \frac{\delta}{2}\right) \times \frac{1}{\left(s + \frac{\delta}{2}\right)^2 - \left(\delta \times \gamma_1 + \frac{\delta^2}{4}\right)}$$

Considering $X(t) = L^{-1}(x)$, we can write:

$$X(t) = X_0 \times L^{-1}\left\{\frac{\left(s + \frac{\delta}{2}\right)}{\left(s + \frac{\delta}{2}\right)^2 - \left(\delta \times \gamma_1 + \frac{\delta^2}{4}\right)}\right\} + X_0 \times \left(\gamma_0 + \frac{\delta}{2}\right) \times L^{-1}\left\{\frac{1}{\left(s + \frac{\delta}{2}\right)^2 - \left(\delta \times \gamma_1 + \frac{\delta^2}{4}\right)}\right\}$$

Or :

$$X(t) = X_0 \times e^{\frac{-\delta}{2}.t} \times \left[\cosh\left(t \times \sqrt{\frac{\delta^2}{4} + \gamma_1 \times \delta}\right) + \frac{\gamma_0 + \frac{\delta}{2}}{\sqrt{\frac{\delta^2}{4} + \gamma_1 . \delta}} \times \sinh\left(t \times \sqrt{\frac{\delta^2}{4} + \gamma_1 \times \delta}\right)\right]$$

Introducing $\gamma_1 = g \times \left(1 + \frac{g}{\delta}\right)$

The equation can be written as:

$$X(t) = X_0 \times \left[\frac{\frac{g + \gamma_0}{2} + \frac{\delta}{2}}{g + \frac{\delta}{2}} \times e^{g.t} + \frac{\frac{g - \gamma_0}{2}}{g + \frac{\delta}{2}} \times e^{-(g+\delta).t}\right] \quad (8)$$

Appendix B: Links between the expressions in continuous time and discrete time

Knowing:

$$\mathrm{NI}_t=\int_{t-1}^{t} X_0\times\left[\frac{\frac{g+\gamma_0}{2}+\frac{\delta}{2}}{g+\frac{\delta}{2}}\times e^{g.t}+\frac{\frac{g-\gamma_0}{2}}{g+\frac{\delta}{2}}\times e^{-(g+\delta).t}\right]dt$$

We can write:

$$\mathrm{NI}_1=X_0\times\left[\frac{\frac{g+\gamma_0}{2}+\frac{\delta}{2}}{g+\frac{\delta}{2}}\times\frac{\left(e^{g}-1\right)}{g}-\frac{\frac{g-\gamma_0}{2}}{g+\frac{\delta}{2}}\times\frac{\left(e^{-(g+\delta)}-1\right)}{(g+\delta)}\right]$$

Or:

$$\mathrm{NI}_1=X_0\times\left[\frac{\frac{g}{2}+\frac{\delta}{2}}{g+\frac{\delta}{2}}\times\frac{\left(e^{g}-1\right)}{g}-\frac{\frac{g}{2}}{g+\frac{\delta}{2}}\times\frac{\left(e^{-(g+\delta)}-1\right)}{(g+\delta)}\right]+X_0\times\gamma_0\times\left[\left(\frac{\left(e^{g}-1\right)}{g}+\frac{\left(e^{-(g+\delta)}-1\right)}{(g+\delta)}\right)\times\frac{1}{2}\times\frac{1}{g+\frac{\delta}{2}}\right]$$

As the same:

$$\mathrm{NI}_2=X_0\times\left[\frac{\frac{g}{2}+\frac{\delta}{2}}{g+\frac{\delta}{2}}\times\frac{\left(e^{2.g}-e^{g}\right)}{g}-\frac{\frac{g}{2}}{g+\frac{\delta}{2}}\times\frac{\left(e^{-2.(g+\delta)}-e^{-(g+\delta)}\right)}{(g+\delta)}\right]+X_0\times\gamma_0\times\left[\left(\frac{\left(e^{2.g}-e^{g}\right)}{g}+\frac{\left(e^{-2.(g+\delta)}-e^{-(g+\delta)}\right)}{(g+\delta)}\right)\times\frac{1}{2}\times\frac{1}{g+\frac{\delta}{2}}\right]$$

Or:

$$\Delta\mathrm{NI}_2=\mathrm{NI}_2-\mathrm{NI}_1=X_0\times\left[\frac{\frac{g}{2}+\frac{\delta}{2}}{g+\frac{\delta}{2}}\times\frac{\left(e^{2.g}-2.e^{g}+1\right)}{g}-\frac{\frac{g}{2}}{g+\frac{\delta}{2}}\times\frac{\left(e^{-2.(g+\delta)}-2.e^{-(g+\delta)}+1\right)}{(g+\delta)}\right]$$
$$+X_0\times\gamma_0\times\left[\left(\frac{\left(e^{2.g}-2.e^{g}+1\right)}{g}+\frac{\left(e^{-2.(g+\delta)}-2.e^{-(g+\delta)}+1\right)}{(g+\delta)}\right)\times\frac{1}{2}\times\frac{1}{g+\frac{\delta}{2}}\right]$$

Considering the matrix M :

$$\begin{bmatrix}\frac{\frac{g}{2}+\frac{\delta}{2}}{g+\frac{\delta}{2}}\times\frac{\left(e^{g}-1\right)}{g}-\frac{\frac{g}{2}}{g+\frac{\delta}{2}}\times\frac{\left(e^{-(g+\delta)}-1\right)}{(g+\delta)} & \left(\frac{\left(e^{g}-1\right)}{g}+\frac{\left(e^{-(g+\delta)}-1\right)}{(g+\delta)}\right)\times\frac{1}{2}\times\frac{1}{g+\frac{\delta}{2}}\\ \frac{\frac{g}{2}+\frac{\delta}{2}}{g+\frac{\delta}{2}}\times\frac{\left(e^{2.g}-2.e^{g}+1\right)}{g}-\frac{\frac{g}{2}}{g+\frac{\delta}{2}}\times\frac{\left(e^{-2.(g+\delta)}-2.e^{-(g+\delta)}+1\right)}{(g+\delta)} & \left(\frac{\left(e^{2.g}-2.e^{g}+1\right)}{g}+\frac{\left(e^{-2.(g+\delta)}-2.e^{-(g+\delta)}+1\right)}{(g+\delta)}\right)\times\frac{1}{2}\times\frac{1}{g+\frac{\delta}{2}}\end{bmatrix}$$

We have:

$$\begin{matrix} \mathrm{NI}_1 \\ \Delta\,\mathrm{NI}_2 \end{matrix} = M \times \begin{matrix} X_0 \\ X_0 \times \gamma_0 \end{matrix}$$

We get:

$$\begin{matrix} X_0 \\ X_0 \times \gamma_0 \end{matrix} = M^{-1} \times \begin{matrix} \mathrm{NI}_1 \\ \Delta\,\mathrm{NI}_2 \end{matrix}$$

Noting:

$$M^{-1} = \begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{vmatrix}$$

It follows:

$$X_0 = m_{11}(g,\delta) \times \mathrm{NI}_1 + m_{12}(g,\delta) \times \Delta\,\mathrm{NI}_2$$
$$X_0 \times \gamma_0 = m_{21}(g,\delta) \times \mathrm{NI}_1 + m_{22}(g,\delta) \times \Delta\,\mathrm{NI}_2$$

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