Finance Bulletin

Finance Bulletin 1:2

On the Properties of Leveraged ETFs

Wided Kout[1]

[1]Université Cergy-Pontoise, UMR 8184, Théorie Economique, Modélisation et Applications (THEMA), 33 Boulevard du Port, F-95011 Cergy-Pontoise Cedex, France.

Corresponding author: wided.kout@u-cergy.fr
Date of submission: 26/03/2019
Date of publication: 20/05/2019

Abstract

In this paper, we examine if, for a successful long-term investment in leveraged ETFs, it is necessary to adjust the level of leverage according to the fluctuations of the financial markets. For this purpose, we illustrate in particular the behavior of the Leverages ETF based on the optimal leverage introduced by Giese (2009). This latter one, which is based on the growth rate expectation, behaves as a function of the prevailing market environment. More precisely, it implies that the investor should use high leverage in low volatility markets and low leverage in high volatility markets. We study also how the degree of leverage depends on the main factor of market environment, namely the volatility of the underlying financial index.

1. Introduction

Financial markets are characterized by their uncertainty as illustrated by the permanent fluctuation of most financial product values. Such risky and often unpredictable environment encourages part of investors to choose products that can enhance equity index performances to maximize their expected wealth. Among the products that have been issued to achieve such objective, are the Leveraged Traded Exchange Funds (LETFs) and the Inverse Exchange Traded Funds (IETFs). In 1997, the company ProFunds was the first to issue a version of the S&P 500 inverse and leveraged index from two mutual funds. In May 2018, the global market for leveraged and inverse ETFs has 263 ETFs including 126 doubles + 2X leverage ETFs and 44 triples + 3X leverage ETFs as well as 66 double inverse ETFs. -2X and 27 triples -3X inverse ETFs. The leverage consists in borrowing on a riskless asset to increase the amount invested on the risky one. Such funds are usually riskier than the standard ones. They aim at double or triple the daily performance of a given financial index. It corresponds to an anticipation of the risky asset rise. When it is the converse, namely borrowing on the risky asset to increase the amount invested on the riskless one, the fund is usually called an "inverse leveraged ETF". Cheng and Madhavan (2009) show how inverse ETFs need to be rebalanced on a daily basis to maintain a constant leverage. Charupat and Miu (2011) state that frequent rebalancing of leveraged ETF portfolios for periods of high volatility is necessary so that leveraged exposure to the tracked index could be maintained. Additionally, Lu *et al.* (2012) show how the longer-term performance diverges from the benchmark through periods of up to one year and how this can lead to wealth destruction, which is also aggravated by higher volatility.

The purpose of this paper is to illustrate the solution proposed by Giese (2009) that allows the long-term holding of these risky products (recall that, for a leverage ETF, "long term" corresponds to a time horizon longer than one year). Indeed, the constant leverage can be turned into conditional leverage that adjusts according to the volatility of the financial markets. As Giese (2009), we refer to this particular investment strategy as a risk-controlled leverage factor. The key question is: does the time-varying leverage factor that is based on estimated future volatility improve the risk-return profile when compared to constant leverage?

This paper is organized as follows. First, Section 2 briefly recalls the main properties of the LETFs when the leverage is constant, and when the risky asset follows a geometric Brownian motion. We illustrate such modeling by using daily data of the EuroStoxx50 from July 12th, 1997 until July 14th, 2017. Section 3 deals with the time varying leveraged ETFs by illustrating the risk controlled leveraged ETFs and by introducing the discrete model of rebalancing. Finally, Section 4 concludes.

2. Price dynamics and statistical properties

2.1. Price dynamics of the Leveraged Exchange Traded Funds (LETF)

A leveraged ETF with value F_t, with an underlying index S_t and a discrete-time rebalancing period can be described as follows (see e.g. Giese, 2009):

$$F_{t+1} = F_t \left[1 + L \frac{S_{t+1} - S_t}{S_t} - (L-1) r \, \Delta t \right] (1)$$

where r denotes the interest rate and where we consider a time period Δt between the rebalancing dates t and $t+1$. Formula (1) indicates that the fund manager borrows *(L - 1)* times the value of the fund at the rate r, to invest L times the net asset value of the fund in the underlying S_t. Due to the frequent rebalancing, the leverage factor is kept constant at the L level. First, we assume that the underlying asset S_t follows a geometric Brownian motion with a constant growth rate u and a constant volatility σ. We have:

$$dS_t = uS_t dt + \sigma S_t dW_t$$

where W_t is a standard Brownian motion.

Then the observed growth rate of the underlying asset is equal to:

$$\mu = u - \frac{\sigma^2}{2} \, (2)$$

The idea is to keep the leverage constant (with a daily rebalancing for example). Using Ito formula, Giese (2009) shows that the value of the leveraged fund is given by:

$$F_t = F_0 \left(\frac{S_t}{S_0}\right)^L \exp\left(-(L-1)rt - \frac{1}{2}L(L-1)\sigma^2 t\right) (3)$$

Thus, the value of the LETF is equal to the payoff $\left(\frac{S_t}{S_0}\right)^L$ times a deterministic function of time.

However, as emphasized in Bertrand and Prigent (2013), there is a significant probability that the stock index price increases while, at the same time, the leveraged fund decreases. This is due to the refinancing cost. Trainor and Baryla (2008) mention also that, on average, leveraged funds meet their specified daily leverage targets but that there is significant volatility in meeting this target on any given day.

2.2. Statistical properties of the Leveraged Exchange Traded Funds (LETF)

The expected value of the fund based on (3) is given by:

$$E[F_t] = F_0 \, exp(Lut - (L-1)rt) \;\; (4)$$

The probability distribution of a percentage of profit or loss x is given by:

$$P_L(x) = P\left(\exp(L\mu t + L\sigma W_t - (L-1)rt - \frac{1}{2}L(L-1)\sigma^2 t) < (1+x)\right)$$

$$= P\left(W_t < \frac{2L(r-\mu)t - 2rt + L(L-1)\,\sigma^2\,t + 2ln(1+x)}{2L\sigma}\right)$$

$$= \phi\left(\frac{2L(r-\mu)t - 2rt + L(L-1)\sigma^2 t + 2ln(1+x)}{2L\sigma}\right) (5)$$

where ϕ(.) denotes the cumulative standard normal distribution illustrated in Figure (1) for different levels L.

In this paper, we perform a back test using the daily data of the EuroStoxx50 from July 12th, 1997 until July 14th, 2017. Therefore, we have a database that consists of 4977 daily data for 20 years. Our aim is to show if the performance of a leveraged investment strategy can be improved by using a variable leverage factor over time that depends on the estimated future volatility. In our case, the drift *u*=0,0805 and the volatility σ=0.2169 are determined from the database described above. We find that the underlying asset has an observable growth rate *u* of 5.7%.

The probability density function of the LETF return is equal to:

$$p_L(x) = P_L'(x) = \varphi\left(\frac{2L(r-\mu)t - 2rt + L(L-1)\sigma^2 t + 2ln(1+x)}{2L\sigma\sqrt{t}}\right)\frac{1}{(1+x)L\sigma\sqrt{t}}$$

Where ϕ=ϕ' is presented on the Figure (1) for different levels of leverage *L*.

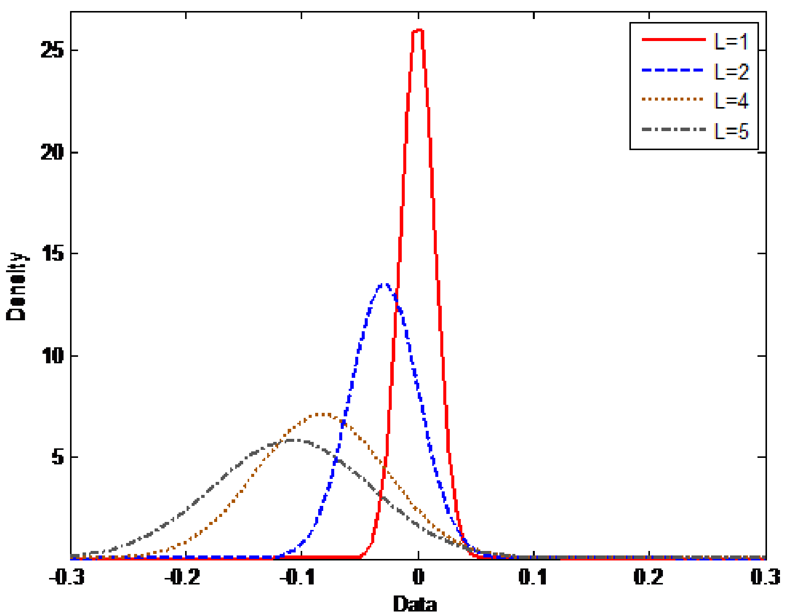

Figure 1: The probability density distribution as function of the leverage.

Figure 1 displays the distribution of the probability of achieving a loss or a gain for *t*=4977 days, *u*=8.05% , σ=21.65% and r=3% for different degrees of leverage : L= 1, L=2, L=4 et L=5.

We note that for high degrees of leverage, the probability of making a loss tends to 1 while the value of the expected bottom tends to infinity.

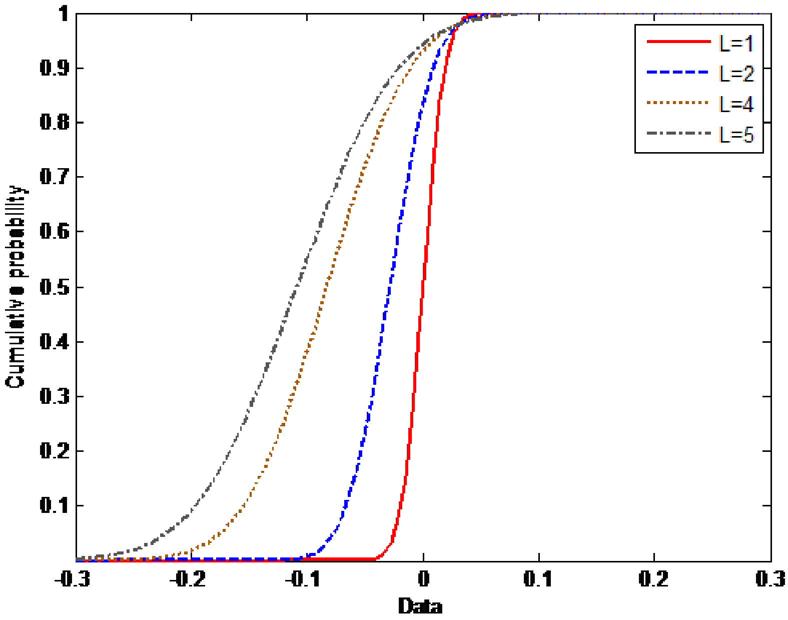

Figure 2: The cumulative distribution function of the fund return.

Figure 2 displays the cdf of the fund return $\left(\frac{F_t}{F_{t-1}} - 1\right)$ based on 4977 daily data and for different degrees of leverage.

It is interesting to note that for $L \rightarrow \infty$ the expected value of the fund tends to infinity (see Relation 4), where the probability of achieving a loss $P_L(x=0)$ tends to 1 according to (Relation 5):

$$P_L(x=0) = \phi\left(\frac{2L(r-\mu)t - 2rt + L(L-1)\sigma^2 t}{2L\sigma\sqrt{t}}\right) \rightarrow 1 \; lorsque \; L \rightarrow \infty \; (6)$$

	L=1	L=2	L=4	L=5
Standard deviation	21.60	43.10	57.20	68.9
Mean (%)	0.02	-2.91	-8.43	-11.04
Skewness	2.62	-5.21	-0.36	0.193
Kurtosis	66.33	165.93	20.81	14.03

Table 1: Four moments for the distribution of daily returns of the LETF with different levels of leverage

From Table 1, we can see that the distributions of the LETF are far from being Gaussian. For example, with a leverage factor L=4, it is significantly different from the normal distribution because its asymmetry coefficient (Skewness) is equal to -0.36 which is lower than 0 (asymmetry to the left) and its flatness coefficient (Kurtosis) is equal to 20.81 which is much higher than 3 (the distribution of this LETF is leptokurtic). Note also that the volatility of the fund is equal to the volatility of the underlying financial index times the leverage level *L*.

As emphasized by Giese (2009), the expected growth rate of the fund can be written as follows:

$$g = \frac{1}{t} E\left[ln\left(\frac{F_t}{F_0}\right)\right] = L\mu - (L-1)r - \frac{1}{2}L(L-1)\sigma^2 (7)$$

which consists of three terms:

- The leveraged return Lu of the underlying asset.
- The refinancing costs – *(L*-1*) r*.
- The volatility term$-\frac{1}{2}L(L-1)\sigma^2$, representing the volatility of losses for a leverage strategy L>1 and the volatility of gains for a leverage strategy *0<L<1*.

Equation (7) indicates that, in the long run, the performance of the leveraged strategy index is *L* times the return of the stock market index minus the refinancing costs minus the term representing the undesirable effects of the rebalancing of the portfolio. The rebalancing losses come from the fact that the performance of the leveraged indices is dependent on the path taken by the underlying index. Importantly, leveraged ETFs with different underlying indices and different leverage factors all share the same characteristics of LT performance that are essentially determined by three parameters namely.

- ✓ The higher the return of the underlying asset, the more attractive the leverage.
- ✓ The higher the interest rate *r*, the less attractive the leverage.
- ✓ The higher the realized average volatility of the underlying, the less attractive the leverage.

We therefore calculate the growth rate *g* for different degrees of leverage *L* in order to find the optimal degree of leverage that maximizes the rate *g*.

L	**1**	**2**	**3**	**4**	**5**
g	**0.057**	**0.036**	**-0.030**	**-0.144**	**-0.305**

Table 2: Growth rate of daily returns of the LETF with different levels of leverage

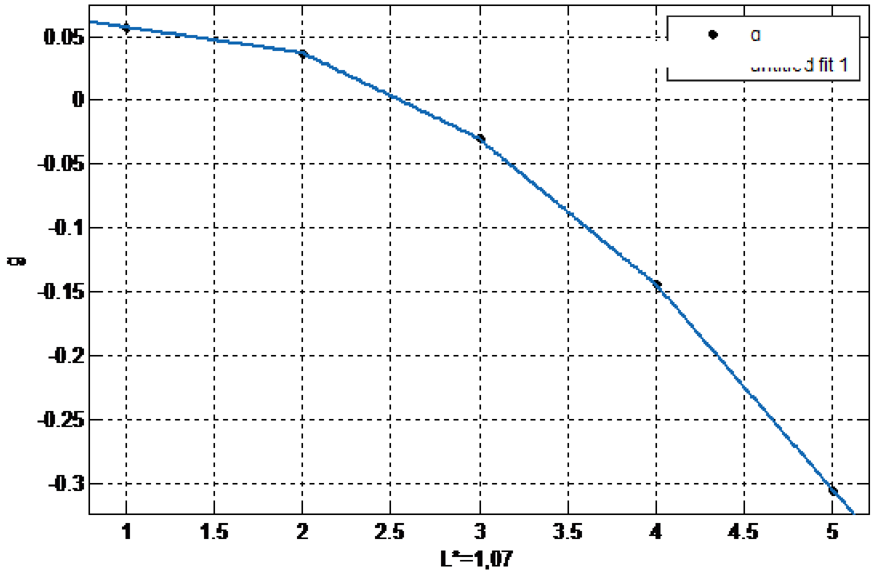

Figure 3: The growth rate *g* as a function of the leverage factor *L*.

Depending on the growth rate of the fund, we get the following modified Sharpe ratio:

$$S = \frac{g - r}{L\sigma} = \frac{\mu - r - \frac{1}{2}(L-1)\sigma^2}{\sigma} \quad (8)$$

The modified Sharpe ratio is the quotient of excess profitability compared to the risk-free rate divided by the total risk of the portfolio. It actually gives the opportunity to calculate the performance of an investment or portfolio compared to that of a risk-free investment. This ratio leads to the fact that:

- If the Sharpe Ratio is <1, the portfolio performs less well than the risk-free investment. You must not invest in it.
- If the Sharpe ratio is between 0 and 1, the portfolio is underperforming the risk undertaken.
- If the Sharpe ratio is greater than 1, the portfolio outperforms the risk-free investment.

L	**1**	**2**	**3**	**4**	**5**
S	**0.124**	**0.016**	**-0.092**	**-0.200**	**-0.309**

Table 3: Modified Sharpe ratio of daily returns of the LETF with different levels of leverage.

The key observation from the modified Sharpe ratio is that for a leveraged strategy (L>1), the modified Sharpe ratio turns out to be always lower than the modified Sharpe ratio of the underlying index which is equal at 0.124. This result is confirmed in our case for which we note that as soon as L>1 the Sharpe ratio starts to deteriorate which is explained by the insufficiency of the returns to face the risk taken by the investor by choosing a leveraged strategy.

$$s = \frac{\mu - r}{\sigma} (9)$$

The reasons for the deterioration in the Sharpe ratio are losses due to rebalancing, which reduces the performance of the leveraged index. The outperformance of leverage funds with the risk-free rate and fund volatility increases linearly with leverage factor L and therefore keeps the Sharpe ratio constant compared to a leverage-less fund L=1. However, the Sharpe ratio is negatively influenced by volatility in the numerator of the Sharpe ratio. Thus, it can be noticed that the use of a leverage factor L >1 indeed increases the expected return on the fund, but the leveraged returns exhibit much higher volatility. In addition, it is important to note that both formulas for the expected growth rate (7) as well as the Sharpe ratio (9) are based on the assumption of log-normal returns while the real-world returns are known to display very low or even negative skewness.

3. The time-varying leveraged ETFs

3.1. Risk-controlled leveraged ETFs

It is interesting to note that the expected growth rate of the leverage fund is a quadratic polynomial function with respect to the leverage factor L. Consequently, it implies the existence of an optimal leverage factor L^* that maximizes the growth rate. We get: (see Giese, 2009)

$$L^* = \frac{1}{2} + \frac{\mu - r}{\sigma^2} \ (10)$$

Relation (10) shows the arbitrage that an investor faces when choosing L between pushing up the growth rate and assuming rebalancing losses due to rising volatilities. In our case, L^* =1.07. We replace L^* from (10) in the function of the growth rate (7) to find the optimal growth rate of the leveraged strategy:

$$g^* = \frac{[\mu - r]^2}{2\sigma^2} + \frac{\mu + r}{2} + \frac{\sigma^2}{8} (11)$$

The maximum expected growth rate g^* in our case study is 18.77%.

3.2. The discrete model of rebalancing leveraged ETFs

In what follows, using financial data, we examine the variation of the optimal leverage and its risk control introduced by Giese (2010). We denote by L_σ this time varying optimal degree of leverage:

$$L_\sigma = \frac{1}{2} + \frac{\mu - r}{\sigma^2} \quad (12)$$

According to equation (12), the optimal way of executing leveraged ETFs is to implement the risk-controlled leverage factor L_σ. The degree of optimal leverage is higher when the volatility is low and the risk premium (*u* - *r*) is high, which is typically the case for bull markets. On the other hand, in the case of bear markets (which are typically characterized by high levels of volatility and low risk premium values) the optimal leverage factor is small. Therefore, for investors using leveraged ETFs for a long-term investment strategy, a risk-controlled methodology is proposed that involves adjusting the leverage of the ETFs according to equation (12).

In this section, we analyze the performance of a leveraged fund over a time period [0, T], which we divide into small intervals $\Delta t = \frac{T}{N}$ and the rebalancing takes place at discrete moments $t_i = i \; {}^{T}/_{N}$. In addition, for each subinterval $[t_i, t_{i+1}]$, we have independent standard random variables x_i which represent the randomness of the returns of the underlying index in the respective time slots. The underlying index return between t_i to t_{i+1} is equal to $u\Delta t + \sigma_i x_{i+1}\sqrt{\Delta t}$, where the variable σ_i represents the volatility in the respective time interval $[t_i, t_{i+1}]$. In addition, we use an interest rate r_i in each time interval. We get:

$$F_T = F_0 \prod_{i=0}^{N-1} \left(1 + L\left(u\Delta t + \sigma_i x_{i+1}\sqrt{\Delta t}\right) - (L-1) r_i \Delta t\right) \quad (13)$$

As a result, using Taylor expansion of order 2, the expected growth rate of the fund is given by:

$$g_{\Delta t} = \frac{1}{T} E\left[ln\left(\prod_{i=0}^{N-1} \left(1 + L\left(u\Delta t + \sigma_i x_{i+1}\sqrt{\Delta t}\right) - (L-1) r_i \Delta t\right)\right)\right]$$

$$= \frac{\Delta t}{T} \sum_{i=0}^{N-1} \left[Lu - (L-1)r_i - \frac{1}{2} L^2 \sigma_i^2 + \frac{1}{3} L^3 \sqrt{\Delta t}\, E\left[\sigma_i^3 x_{i+1}^3\right]\right] + \; O\left(\Delta t^{3/2}\right)$$

$$= \underset{\textit{Continuous term}}{\underset{\downarrow}{L(u - \bar{\sigma}\hat{}2/2) - (L-1)\,\bar{r} \; - 1/2\, L(L-1)\bar{\sigma}\hat{}2}} \; + \; \underset{\textit{Correction term}}{\underset{\downarrow}{1/3\, L\hat{}3\, \bar{\sigma}\hat{}3\, \kappa\_\Delta \sqrt{(\Delta t\,)}}} \; + \; O\left(\Delta t^{3/2}\right) \quad (14)$$

where E represents the expectation operator, $\bar{\sigma} = \left(\frac{1}{N}\sum_{i=0}^{N-1}\sigma_i^2\right)^{\frac{1}{2}}$, $\bar{r} = \frac{1}{N}\sum_{i=0}^{N-1} r_i$ which represent respectively the average values of volatility and interest rate and $K_\Delta = \frac{1}{N}\sum_{i=0}^{N-1}\left({\sigma_i^3}/{\bar{\sigma}^3}\right)E\left[x_i^3\right]$ is the skewness of the random variables x_i. The key question is whether the correction term has a significant influence on the average performance of leveraged ETFs, where Δt is each daily, weekly or monthly rebalancing period. As the correction term increases in the rebalancing period, we limit our analysis to monthly rebalancing. We can see that, for a leverage factor equal to four, the correction term is clearly less than one basis point per year. This implies that the rebalancing frequency has a negligible influence on leveraged ETFs in the real world. We conclude that the characteristics of the long-term performance of Leveraged ETFs are accurately described with daily, weekly and monthly rebalancing.

Considering volatility on a regular basis, we illustrate the advantages of this methodology by simulating a leveraged ETF based on the returns of the EUROSTOXX 50 stock index. In order to avoid transaction costs related to high turnover rates, we proceed to the adjustment only once a month. More precisely, on the first trading day of each month, we evaluate the optimal degree of leverage according to formula (3). Our database used in this study is composed of the monthly returns of the EUROSTOXX 50 index for ten years from January 31, 2007 to January 31, 2017. For volatility, we use the monthly historical volatility of this index for the same period. Then regarding the interest rate, we use the monthly Overnight USD Libor rates to calculate the leveraged versions of the underlying index. Then, the growth rate μ achieved by the underlying index EUROSTOXX 50 in the long run is supposed to be constant but in the medium term it is interesting to detect possible bear markets through this rate.

In what follows, we round up the optimal leverage to integers 0,1,2...

- If L_σ is equal to zero, then during the whole month the investment amount is invested in the USD Libor market without any exposure to risk.
- If L_σ is equal to one, then the investment amount is invested in an unleveraged ETF using the underlying EUROSTOXX 50 asset.
- If L_σ is equal to two or three, then the investment strategy will be invested in a double or triple leveraged ETF using the underlying EUROSTOXX 50 asset.

According to Figure (4), we can see that the leverage factor L varies in the opposite direction of the volatility. For example, the lowest leverage level of *0.62* that was realized on October 01, 2008 corresponds to the highest volatility during the 10 years of the study, which is equal to 60%. That is quite logical because this period corresponds to the financial crisis of 2008 during which the markets became volatile and bearish. On the other hand, the higher leverage level of 3.75≅4 achieved in October 2013 corresponds to the lower volatility 14% which is consistent with the interpretations of Equation (12). Then Figure (5) compares the performance of the leveraged risk-controlled strategy for the underlying Eurostoxx 50 index and a constant leverage ETF of 2. In this figure, we note that the two strategies with constant and conditional leverage almost match. Knowing that the average of monthly leverage levels - applied monthly depending on monthly volatility - is equal to 1.7 which can be rounded to 2 which is leveraged strategy with a constant leverage factor. From the end of 2007, which corresponds to the beginning of the subprime crisis, constant and time-varying leveraged strategies begin to underperform the underlying index up to the year 2017.

In this case, the most interesting leverage to apply is as it was found with equation (10) in the single-active model equal to 1.07 corresponding to fixed L^* that is close to 1. In other words, it is more interesting to replicate the underlying without taking risks.

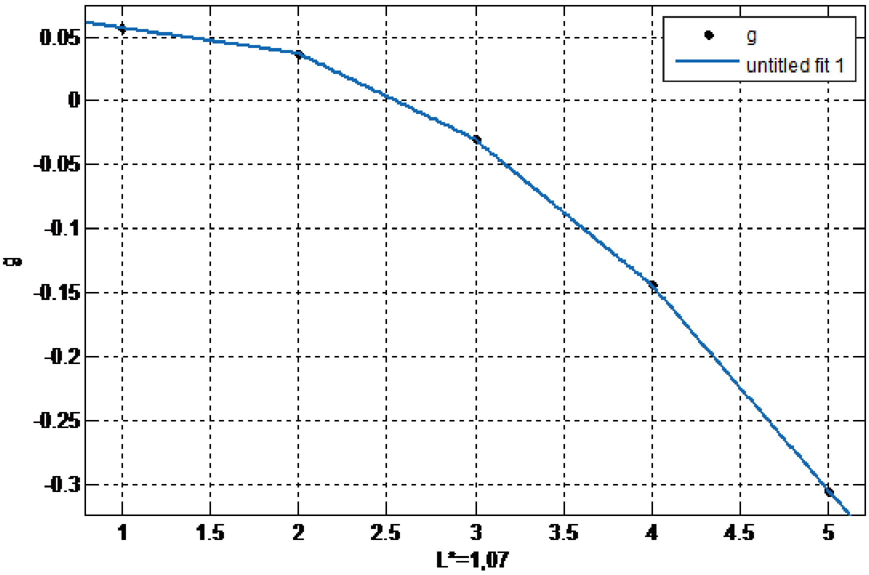

Figure 4: The evolution of the leverage according to the volatility of the risky asset.

Figure 5: The historical performance of the risk-controlled strategy relative to the returns of the underlying index and those of the leveraged fund.

4. Conclusion

Leveraged or inverse leveraged ETF are becoming increasingly popular in the financial industry. Leveraged ETF offer an advantage and disadvantage for the investor. The main benefit of the daily rebalancing strategy is that, in the case of positive market movements, exposure to risk through investment in an underlying asset increases on a daily basis to improve profits. When markets change in the adverse direction, exposure is reduced on a daily basis to protect investors from serious losses. On the other hand, the main disadvantage is the occurrence of losses due to the volatility of daily rebalancing. Therefore, the long-term performance of these products depends on the trade-off between leveraged returns, which are linear in the chosen leverage factor and the loss due to volatility, which is quadratic in the leverage factor. Note that usually the leveraged ETFs use a constant degree of leverage that does not respond to the market environment. In this paper, we have discussed the benefit of introducing a time varying leverage factor which is adjusted according to the concept of optimal leverage in order to limit the losses investors face in bear markets. Our empirical results are rather disappointing showing that the optimal leverage introduced by Giese (2009) is quite close to 2, even during the financial crisis. Such feature suggests the introduction of another time-varying leverage more based on risk measures such as quintile or expected shortfalls in the line of Ben Ameur and Prigent (2014).

References

Ben Ameur H., Prigent J.-L., 2014. Portfolio insurance: Gap risk under conditional multiples. *European Journal of Operational Research*, 236: 238-253

Bertrand P., Prigent J.-L., 2013. Analysis and comparison of leveraged ETFs and CPPI-type leveraged strategies. *Finance*, 34: 73-116

Charupat N., Miu P., 2011. The pricing and performance of leveraged exchange-traded funds. *Journal of Banking and Finance*, 35: 966-977

Cheng M., Madhavan A., 2009. The Dynamics of Leveraged and Inverse-Exchange Traded Funds. *Journal of Investment Management*

Giese G., 2009. On the performance of leveraged and optimally leveraged investment funds. *Available at SSRN*

Giese G., 2010. A dynamic model for leveraged funds. *Risk Magazine*

Lu L., Wang J., Zhang G., 2012. Long term performance of leveraged ETFs. *Financial Services Review*, 21: 63-80

Trainor W., Baryla E., 2008. Leveraged ETFs: A risky double that doesn’t multiply by two. *Journal of Financial Planning*, 21: 48-55